Continuum Processes

(in High-Energy Astrophysics)

3rd X-ray Astronomy school

Wallops Island May 12-16 May

Ilana HARRUS

(USRA/NASA/GSFC)

Bremsstrahlung

What is Bremsstrahlung?

(and why this bizarre name of “braking radiation”?)

Historically noted in the context of the study of electron/ion interactions. Radiation of EM waves because of the acceleration of the electron in the EM field of the nucleus.

And … when a particle is accelerated it radiates .

Bremsstrahlung

Important because for relativistic particles, this can be the dominant mode of energy loss.

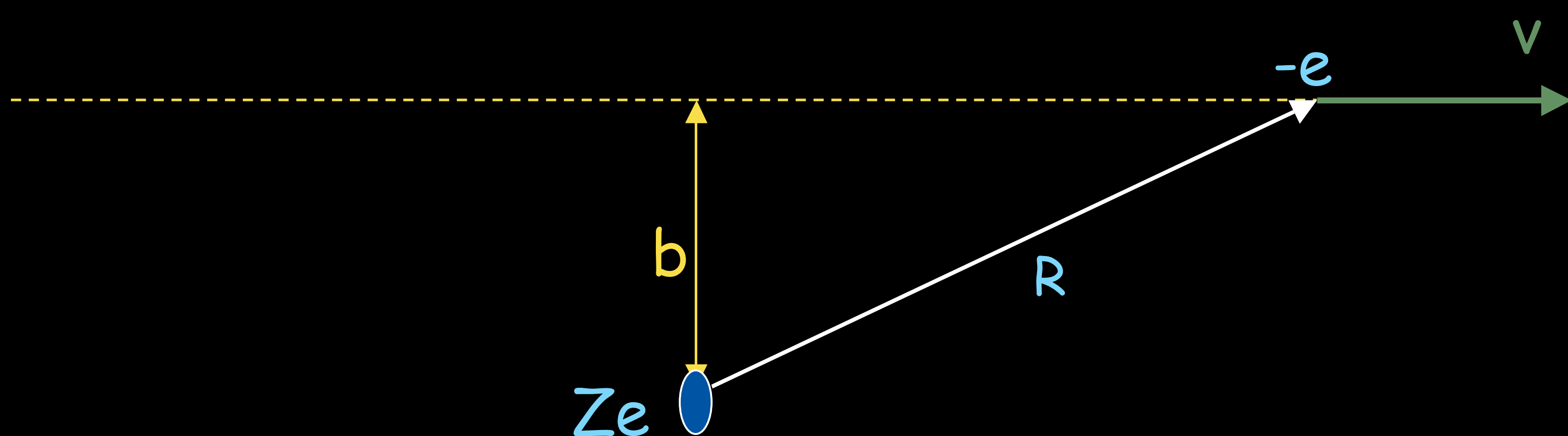
Whenever there is hot ionized gas in the Universe, there will be Bremsstrahlung emission.

Provide information on both the medium and the particles (electrons) doing the radiation.

The complete treatment should be based on QED ===> in every reference book, the computations are made "classically" and modified ("Gaunt" factors) to take into account quantum effects.

Bremsstrahlung (*non relativistic*)

Use the dipole approximation (fine for electron/nucleus bremsstrahlung)---



Electron moves mainly in straight line--

$$\Delta v = Ze^2/m_e \int (b^2+v^2t^2)^{-3/2}\, b\,dt = 2Ze^2/mbv$$

Electric field: $E(t) = Ze^3 \sin\theta / m_e c^2 R (b^2+v^2t^2)$

Bremsstrahlung (for one NR electron)

(using Fourier transform)

$$E(\square) = Ze^3 \sin\square / m_e c^2 R \; \square/bv \; e^{-b\square/v}$$

Energy per unit area and frequency is:

$$dW/dAd\square = c|E(\square)|^2$$

So that integrated on all solid angles:

$$dW(b)/d\square = 8/3\square \, (Z^2 e^6 / m_e{}^2 c^3) \, (1/bv)^2 \, e^{-2b\square/v}$$

Bremsstrahlung

For a distribution of electrons in a medium with ion density n_i. Electron density n_e and same velocity v.

Emission per unit time, volume, frequency:

$$dW/dVdtd\omega = n_e n_i 2\pi v \int_{bmin} dW(b)/d\omega \; bdb$$

Approximation: contributions up to b_{max}

This implies (after integration on b)

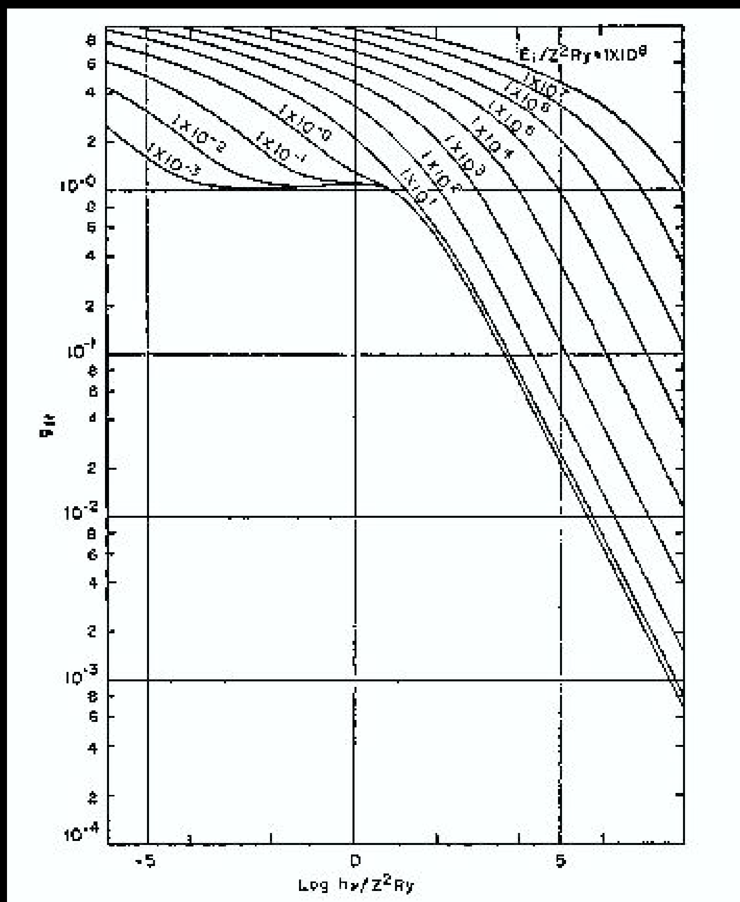
$$dW/dVdtd\omega = (16e^6/3m_e^2vc^3)\, n_e n_i Z^2 \ln(b_{max}/b_{min})$$

with $b_{min} \sim h/mv$ and $b_{max} \sim v/\omega$

Bremsstrahlung

If QED used, the result is:

$$dW/dVdtd\nu = (16\pi e^6/3^{3/2} m_e^2 v c^3)\, n_e n_i Z^2\, g_{ff}(v,\nu)$$



Karzas & Latter, 1961, ApJS, 6, 167

Bremsstrahlung

Now for electrons with a Maxwell-Boltzmann velocity distribution. The probability dP that a particle has a velocity within d^3v is:

$$dP \quad e^{-E/kT}\, d^3v \quad v^2 e^{(-mv2/2kT)} dv$$

Integration limits: $1/2\, mv^2 \gg h□$ (Photon discreteness effect) and using $d□=2□d□$

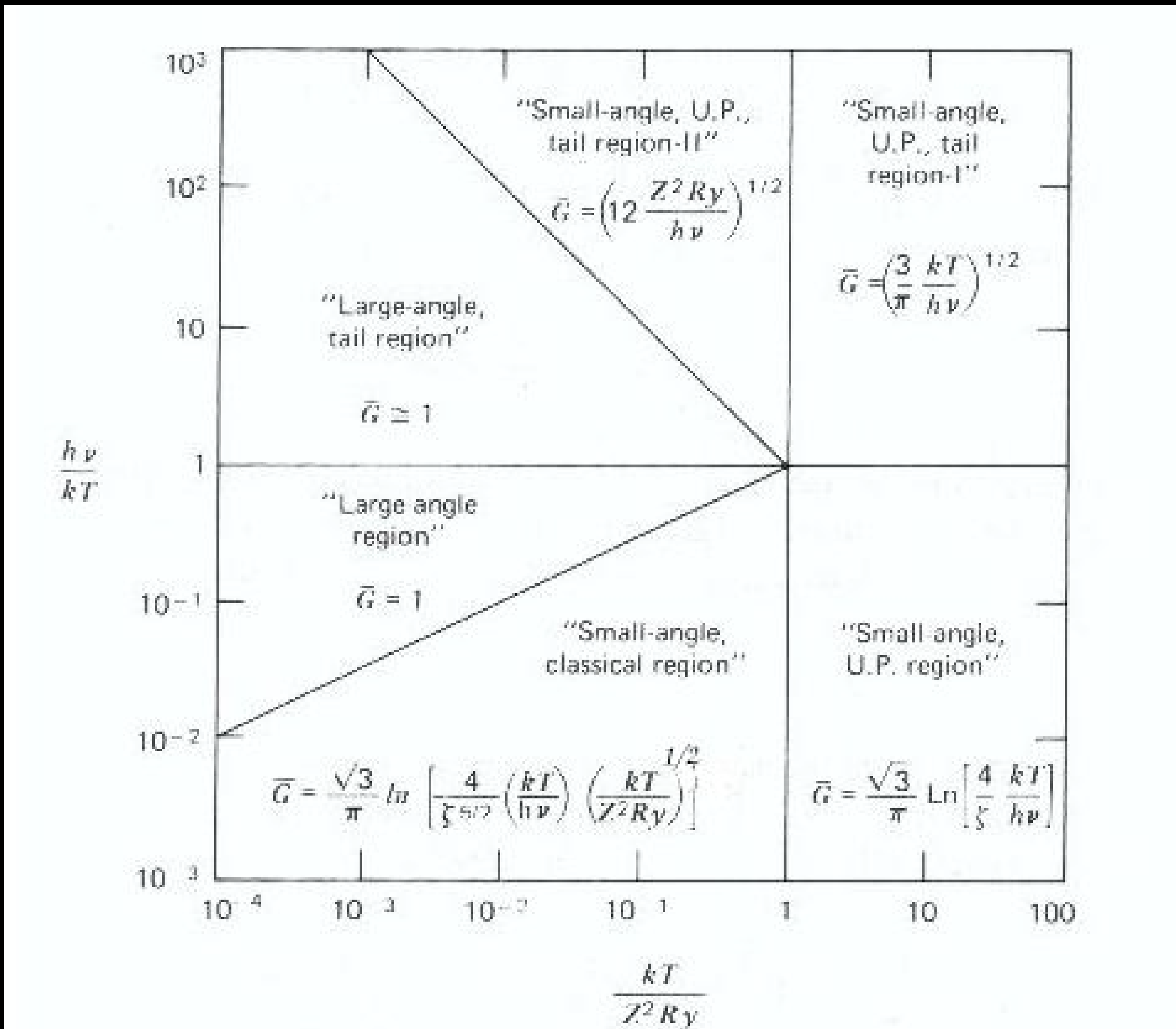
$dW/dVdtd□ =$

$$(32□e^6/3m_e c^3)\,(2□/3kTm_e)^{1/2}\, n_e n_i Z^2\, e^{(-h□/kT)} \langle g_{ff}\rangle ==$$

$$6.8\ 10^{-38}\ T^{-1/2} n_e n_i Z^2\, e^{(-h□/kT)} \langle g_{ff}\rangle \text{ erg s}^{-1}\text{ cm}^{-3}\text{ Hz}^{-1}$$

$\langle g_{ff}\rangle$ is the velocity average Gaunt factor

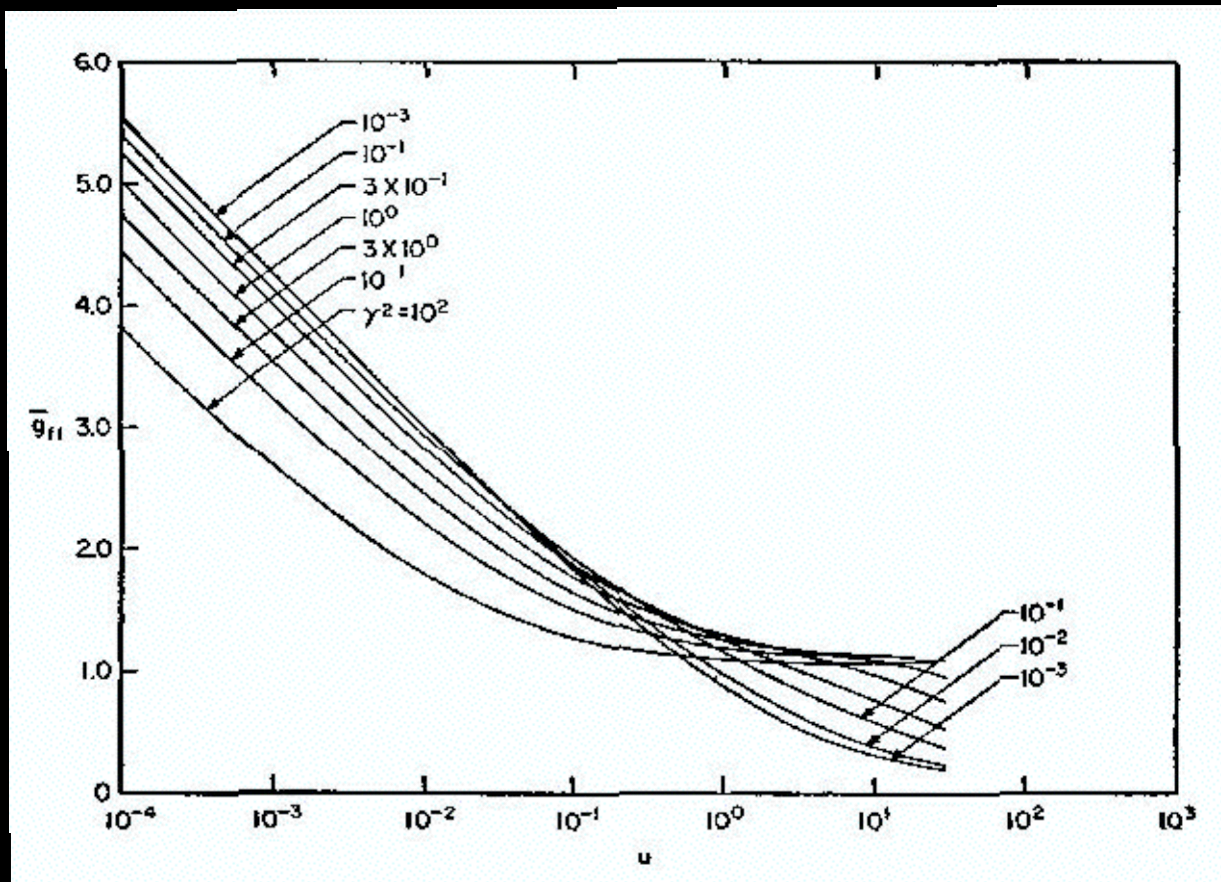
Bremsstrahlung



Approximate analytic formulae for $\langle g_{ff} \rangle$

From Rybicki & Lightman Fig 5.2 (corrected) -- originally from Novikov and Thorne (1973)

Bremsstrahlung



$u = h\nu/kT$;

$\gamma^2 = Ry\, Z^2/kT$

$= 1.58 \times 10^5\, Z^2/T$

Numerical values of $\langle g_{ff} \rangle$.

From Rybicki & Lightman Fig 5.3 -- originally from Karzas & Latter (1961)

When integrated over frequency:

$$dW/dVdt = (32\pi e^6/3hm_e c^3)\,(2\pi kT/3m_e)^{1/2}\, n_e n_i Z^2 \langle g_B \rangle$$

$$== 1.4\times10^{-27}\, T^{1/2} n_e n_i Z^2 \langle g_B \rangle \text{ erg s}^{-1} \text{ cm}^{-3} \quad (1+4.4\times10^{-10}T)$$

Cyclotron/Synchrotron Radiation

Radiation emitted by charge moving in a magnetic field.

First discussed by Schott (1912). Revived after 1945 in connection with problems on radiation from electron accelerators, …

Very important in astrophysics: Galactic radio emission (radiation from the halo and the disk), radio emission from the shell of supernova remnants, X-ray synchrotron from PWN in SNRs…

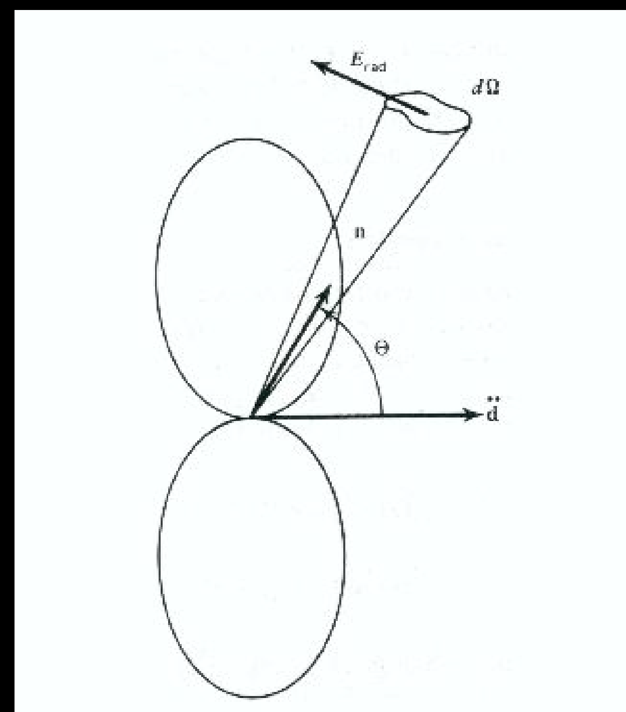
Cyclotron/Synchrotron Radiation

As with Bremsstrahlung, complete (rigorous) derivation is quite tricky.

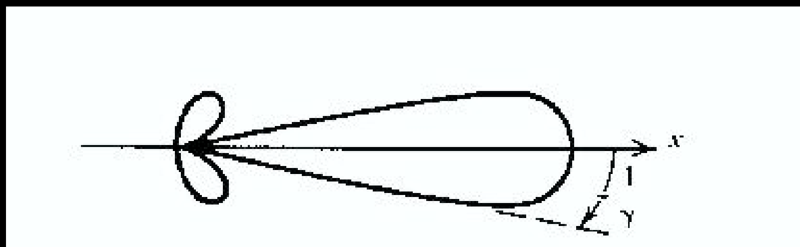
First for a non-relativist electron: frequency of gyration in the magnetic field is $\square_L$ =eB/mc

= 2.8 B_{1G} MHz (Larmor)

Frequency of radiation == $\square_L$



Synchrotron Radiation



Angular distribution of radiation (acceleration $\perp$ velocity).

Rybicki & Lightman

Because of relativistic effects: beaming and $\Delta\theta \sim 1/\gamma$

Gyration frequency $\omega_B = \omega_L/\gamma$

Observer sees radiations for duration $\Delta t \lll T = 2\pi/\omega_B$

This means that the spectrum includes higher harmonics of ω_B.

Maximum is at a characteristic frequency which is:

$$\omega_c \sim 1/\Delta t \sim \gamma^2 e B_\perp / mc$$

Synchrotron Radiation

Total emitted radiation is:

$P= 2e^4 B_{□}^2/3m^2c^3$ □^{2}□^{2} = 2/3 r_0^2 c □^{2} $B_{□}^2$ when □>>1

Or P □ □^{2} c □$_T$ U_B $\sin^2$□ (U_B is the magnetic energy density)

P~1.6x10^{-15} □^{2} B^2 $\sin^2$□ erg s^{-1}

Life time of particle of energy □ □/P ~ 20/□B^2 yr

Example of Crab-- Life time of X-ray producing electron is about 20 years.

P □ 1/m^2 : synchrotron is negligible for massive particles.

Synchrotron Radiation

The computation of the spectral distribution of the total radiation from one UR electron: (computation done in both polarization directions -- parallel and perpendicular to the direction of the magnetic field).

P□(□) = (3e^3/4□mc^2) B sin□ [F(x)+G(x)] ; x=□/□c

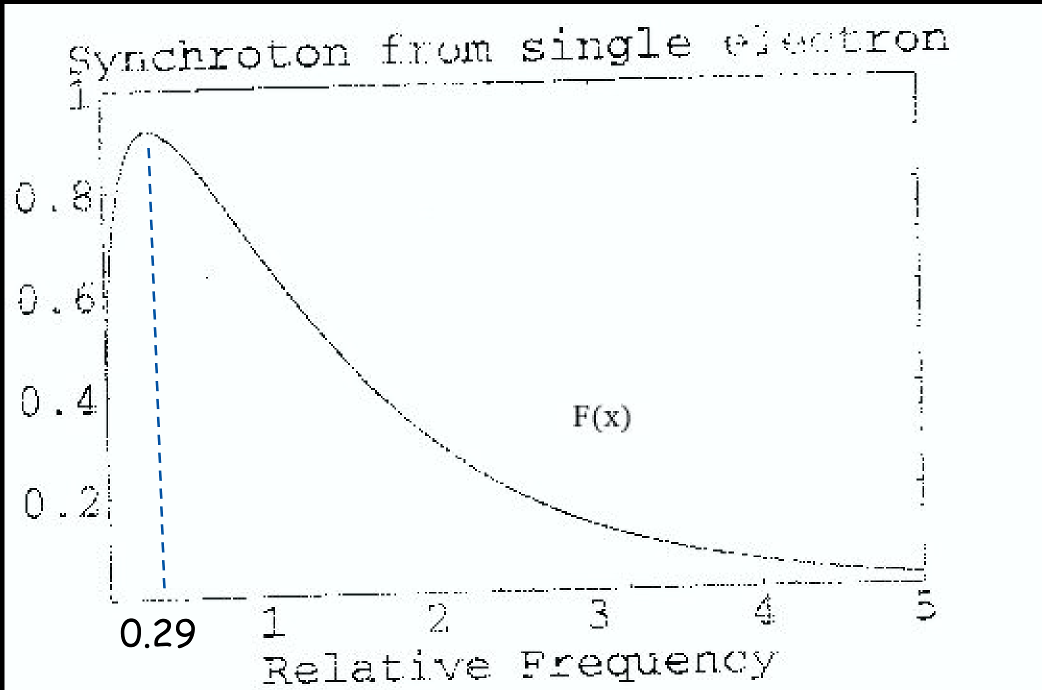
P II (□) =(3e^3/4□mc^2) B sin□ [F(x)-G(x)]

Where F(x)==x$\int^{\infty}_{x}$ K $_{5/3}$(y)dy ; G(x)==x K $_{5/3}$(x) and K modified Bessel function

and □$_c$ = 3/2□^{2} □$_L$sin□

Total emitted power per frequency:

P(□)=(3e^3/2□mc^2) B sin□ F(□/ □$_c$)


Synchroton from single electron
F(x)
0.29
Relative Frequency

Synchrotron Radiation

Hypothesis: Energy spectrum of the electrons between energy E1 and E2 can be approximated by a power-law --

$N(E)=KE^{□□}\, dE$ (isotropic, homegeneous).

Number of e^- per unit volume, between E and E+dE (in arbitrary direction of motion)

Intensity of radiation in a homogeneous magnetic field:

$I(□,k)=(\ 3/□+1)\ □(3□-1/12)\ □(3□+19/12)\ e^3/mc^2\ (3e/2□m^3c^5)^{(□-1)/2}$
$K\ [B\ \sin□]^{(□+1)/2}\ □^{-(□-1)/2}$

Average on all directions of magnetic field (for astrophysical applications).
L is the dimension of the radiating region

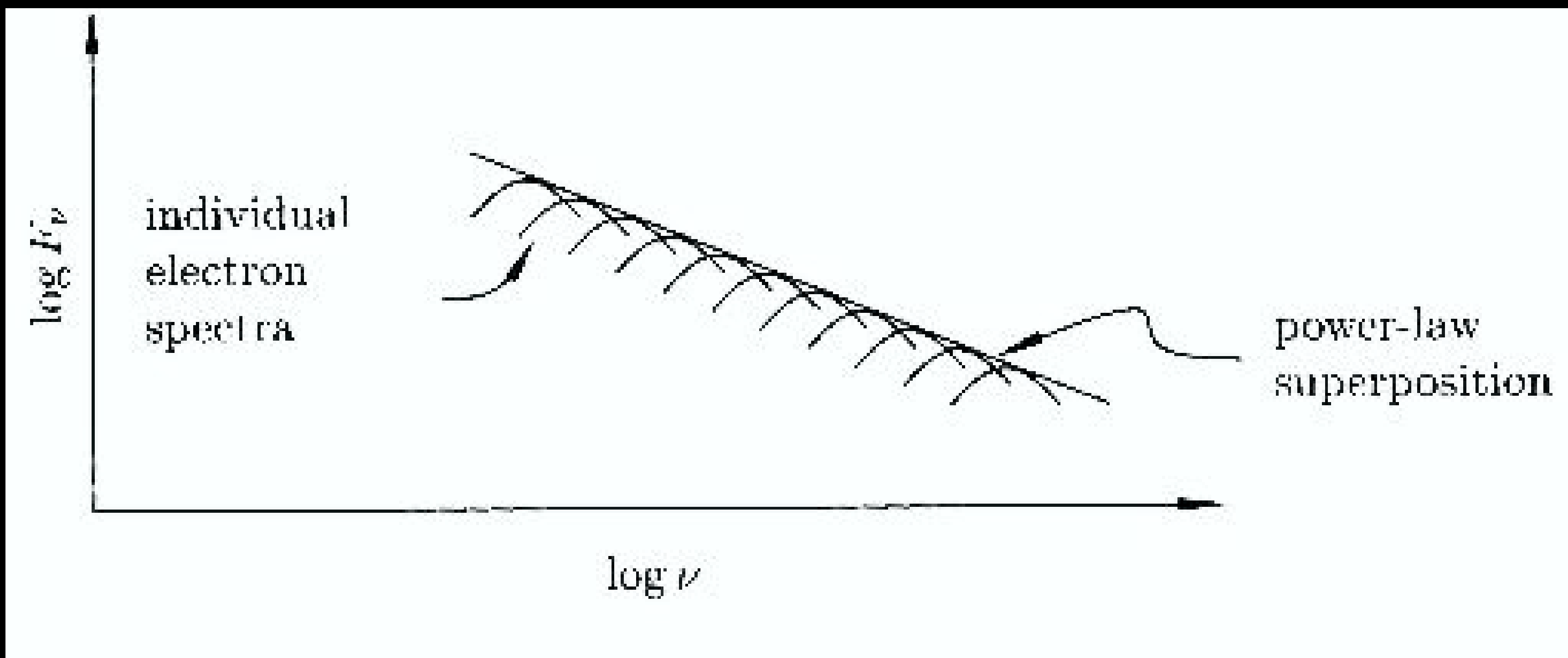
$I(□)= a(□)\ e^3/mc^2\ (3e/4□m^3c^5)^{(□-1)/2}\ \ B^{(□+1)/2}\ K\ L\ □^{-(□-1)/2}$ erg $cm^{-2}\ s^{-1}\ ster^{-1}\ Hz^{-1}$

where

$a(□) = 2^{(□-1)/2}\ \ (3/□)□(3□-1/12)\ □(3□+19/12)\ □(□+5/4)/[8(□+1)\ □(□+7/4)]$

Synchrotron Radiation

If the energy distribution of the electrons is a power distribution



Synchrotron Radiation

Estimating the two boundaries energies E_1 and E_2 of electrons radiating between ν_1 and ν_2.

$$E_1(p) \leq mc^2 \,[4\pi mc\nu_1/3eBy_1(p)]^{1/2} = 250\,[\nu_1/By_1(p)]^{1/2}\ \mathrm{eV}$$

$$E_2(p) \leq mc^2 \,[4\pi mc\nu_2/3eBy_2(p)]^{1/2} = 250\,[\nu_2/By_2(p)]^{1/2}\ \mathrm{eV}$$

$y_1(p)$ and $y_2(p)$ are tabulated (or you can compute them yourself..).

If interval $\nu_2/\nu_1 \ll y_1(p)/y_2(p)$ or if $p<1.5$ this is only rough estimate

Synchrotron Radiation

Expected polarization:

$(P\square(\square) - P_{\parallel}(\square))/(P\square(\square) + P_{\parallel}(\square)) == (\square+1)/(\square+ 7/3)$

can be very high (more than 70%).

Synchrotron Self-Absorption

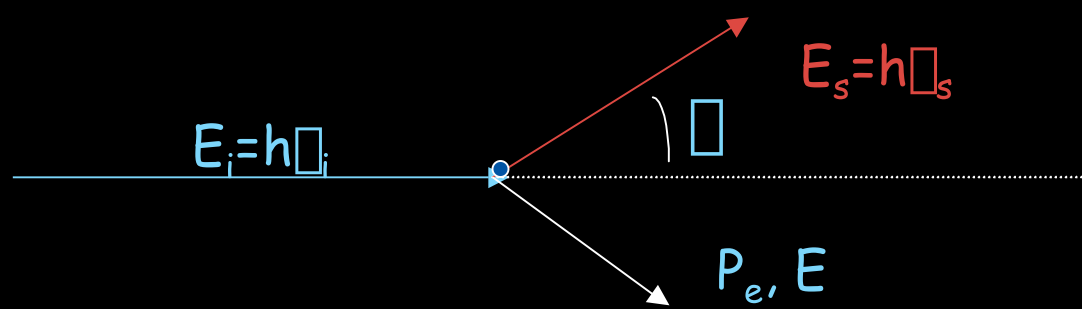
A photon interacts with a charged particle in a magnetic field and is absorbed (energy transferred to the charge particle).
This occurs below a cut-off frequency

The main result is:
For a power-law, the optically thick spectrum is proportional to $B^{-1/2}\nu^{5/2}$ independent of the spectral index.

====> break frequency

Compton/Inv Compton Scattering

For low energy photons (h□ << mc^2), scattering is classical Thomson scattering (E_i=E_s; $□_T$ = 8□/3 r_0^2)



More general: E_s=E_i (1+E_i(1-cos□)/mc^2)$^{-1}$ or

$□_s$-$□_i$=$□_c$(1-cos□) ($□_c$ =h/mc)

This means that E_s is always smaller than E_i

Even more general: (Klein-Nishina)

d□/d□ = 1/2 r_0^2 y^2(y+1/y -$\sin^2$□) with y=E_s/E_i

Compton/Inv Compton Scattering

If electron kinetic energy is large enough, energy transferred from electron to the photon: Inverse Compton

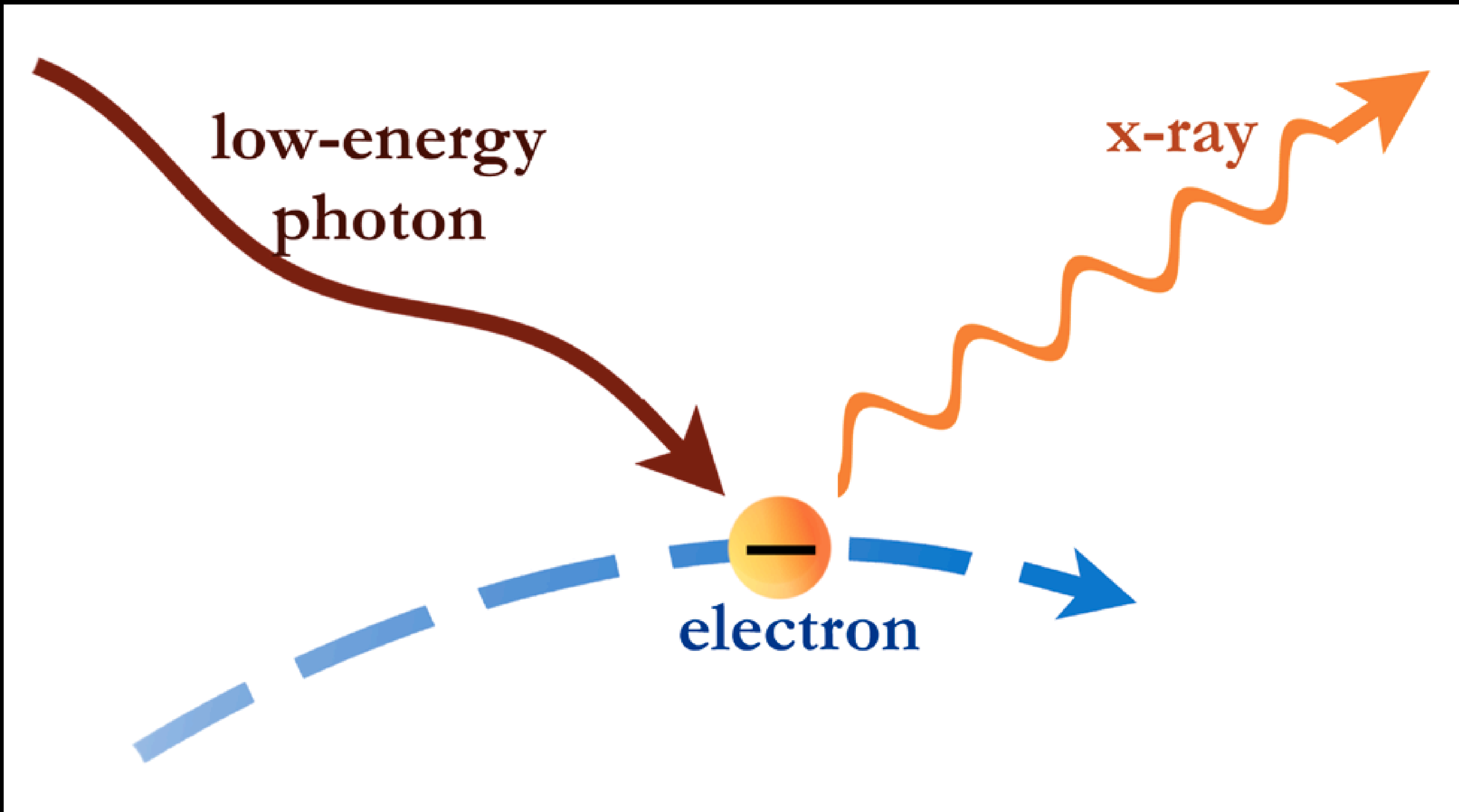
One can use previous formula (valid in the rest frame of the electron) and then Lorentz transform.

So : $E_i^{foe} = E_i^{lab} \square (1 - \square \cos \square)$ then E_i^{foe} becomes E_s^{foe} and $E_s^{lab} = E_s^{foe} \square (1 + \square \cos \square')$

This means that $E_s^{lab} \quad E_i^{lab} \square^2$

The boost can be enormous!

Inverse Compton Scattering



Compton/Inv Compton Scattering

The total power emitted:

$$P_{compt} = 4/3\ \sigma_T c \beta^2 \gamma^2 U_{ph}[1-f(\gamma E_i^{lab})] \sim 4/3\ \sigma_T c \beta^2 \gamma^2 U_{ph}$$

And U_{ph} is the initial photon energy density

We had $P_{sync} \propto \gamma^2\, c\ \sigma_T\ U_B$

In fact : $P_{sync}/P_{compt} = U_B / U_{ph}$

Synchrotron == inverse Compton off virtual photons in the magnetic field.

Both synchrotron and IC are very powerful tools ===> direct access to magnetic and photon energy density.

Not covered

- Thermal bremsstrahlung absorption (energy absorbed by free moving electrons)
- Black body radiation
- Transition radiation *(often not mentioned)*

Books and references

- Rybicki & Lightman "Radiative processes in Astrophysics"
- Longair "High Energy Astrophysics"
- Shu "Physics of Astrophysics"
- Tucker "Radiation processes in Astrophysics"
- Jackson "Classical Electrodynamics"
- Pacholczyk "Radio Astrophysics"
- Ginzburg & Syrovatskii "Cosmic Magnetobremmstrahlung" 1965 Ann. Rev. Astr. Ap. 3, 297
- Ginzburg & Tsytovitch "Transition radiation"